UNIVERSITY OF MSILA **2023/2024**

SECOND SERIES IN ALGEBRA 1

EXERCISE01

We consider the subsets of *N*.

$A = \{1,2,3,4,5,6,7\}$,$B = \{1,3,5,7\}$,$C = \{2,4,6\}$,$D = \{3,6\}$

Determine

$B \cap D$, $C \cap D$, $B \cup C$, $C \cup D$, $C \Delta D$, $C_E B$ $C_E C$ where $E = A$

EXERCISE02

Let E be a set, A, B, C, and D be subsets of E. Show that

1°) $A \cap B = \phi \iff A \subset C_E B.$

2°) $A \subset B \iff A \cap C_E B = \phi.$

3°) $A \subset B \iff C_E(B) \subset C_E(A)$

4°) $(A \backslash B) \cap (A \cap B) = \phi.$

5°) $(A \backslash B) \cup (A \cap B) = A.$

6°) $A \backslash B = C_E B \backslash C_E A = A \cap C_E B$

7°) $A \cap B = A \cap C \iff A \cap C_E B = A \cap C_E C.$

8°) $A \Delta B = (A \cup B) \backslash (A \cap B).$

9°) $(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D)$

10°) $A \subset B \iff \mathcal{P}(A) \subset \mathcal{P}(B)$

EXERCISE03

Let $g : E \longrightarrow F$ be a function. Let A and B be two subsets of F. Prove that:

1. $f(A \cap B) \subset f(A) \cap f(B).$
2. $f(A \cup B) = f(A) \cup f(B)$

3. $g^{-1}(A \cup B) = g^{-1}(A) \cup g^{-1}(B)$

4. $g^{-1}(A \cap B) = g^{-1}(A) \cap g^{-1}(B)$

5. $g^{-1}(C_F B) = C_E g^{-1}(B)$

EXERCISE04

Consider the function f defined by

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto f(x) = \frac{2x}{1+x^2}$$

1. Is f injective? Surjective?

2. Show that $f(\mathbb{R}) = [-1, 1]$.

3. Show that the function g defined by

$g : [-1, 1] \longrightarrow [-1, 1]$

$x \longmapsto g(x) = f(x)$

is a bijection and find its inverse function g^{-1}