Physics 01: Mechanics of point particle. **University Year 2023-2024**

Solutions of tutorial N° 05 Practice: Energy and work

EXERCISE 01

Net force $\vec{F} = \vec{F_1} + \vec{F_2} = (\vec{\imath} + 2\vec{\jmath} + 3\vec{k}) + \overrightarrow{(4\imath - 5\vec{\jmath} - 2\vec{k})} = \overrightarrow{5\imath} - 3\vec{\jmath} + \vec{k}$

Displacement $\overrightarrow{r_{21}} = \vec{r_1} - \vec{r_2} = (20\vec{\imath} + 15\vec{\jmath}) - 7\vec{k} = 20\vec{\imath} + 15\vec{\jmath} - 7\vec{k}$ cm

Work done $W = \vec{F}.\overrightarrow{r_{21}} = \overrightarrow{(5\imath} - 3\vec{\jmath} + \vec{k}).(0.20\vec{\imath} + 0.15\vec{\jmath} - 0.07\vec{k}) = 0.48\,J$

Or:

$$\boldsymbol{W = \int_0^{0.02} F_x.\,dx + \int_0^{0.15} F_y.\,dy + \int_{0.07}^{0} F_z.\,dz}$$

$$W = \int_0^{0.02} 5.\,dx - \int_0^{0.15} 3.\,dy + \int_{0.07}^{0} dz$$

$$W = 5\,x\Big|_0^{0.02} - 5\,y\Big|_0^{0.15} + z\Big|_{0.07}^{0}$$

$$\boldsymbol{W = 0.48\,J}$$

EXERCISE 02

a- $E_p(x) = 5x^2 - 4x^3$

$$\vec{F} = -\overrightarrow{grad}\,E_p(x) = -\frac{\partial E_p(x)}{\partial x}\vec{\imath} = -\frac{dE_p(x)}{dx}\vec{\imath} = (-10x + 12x^2)\vec{\imath}$$

b- For equilibrium: $\boldsymbol{\frac{dE_p(x)}{dx} \Rightarrow F = 0}$

$-10x + 12x^2 = 0$

$(-10 + 12x)x = 0 \Rightarrow \quad x = 0$ or $x = \frac{5}{6}$ m

To know the stable equilibrium position and unstable equilibrium position, we study the sign of $\frac{d^2E_p(x)}{dx^2}$

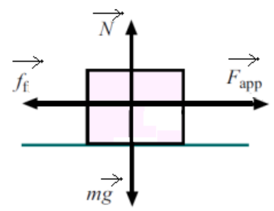
$$\frac{d^2E_p(x)}{dx^2} = \frac{dF}{dx} = 24x - 10$$

$\frac{dF}{dx}\big|_{x=0} = (24x - 10)\big|_{x=0} = -10 < 0 \Rightarrow$ the position $x = 0$ is stable.

$\frac{dF}{dx}\big|_{x=\frac{5}{6}} = (24x - 10)\big|_{x=\frac{5}{6}} = +10 > 0 \Rightarrow$ the position $x = \frac{5}{6}$ is unstable

EXERCISE 03

m =40 kg , V_0= 0 m/s, x= 5 m , F= 130 N, μ= 0.3.



1- The work done by the applied force.

$W(\vec{F}_{app}) = \int_A^B \vec{F}_{app}.\,\overrightarrow{dr} = \int_A^B F_{app}.\,dr.\cos\theta = F_{app}.\,d.\cos 0 = 130 \times 5$

$$\Rightarrow \boldsymbol{W(\vec{F}_{app}) = 650\ J}$$

2- The energy lost due to friction.

$$W(\vec{F}_f)=\int_A^B \vec{F}_f.\overrightarrow{dr}=\int_A^B F_f.\,dr.\cos\theta=F_f.\,d.\cos 180$$

$$F_f=\mu N=\mu\, mg \quad (N=mg)$$

$$W(\vec{F}_f)=-\mu\, mg\, d$$

$$\Rightarrow W(\vec{F}_f)=-0.3\times 40\times 9.8\times 5 \Rightarrow \mathbf{W(\vec{F}_f)=-590\ J}$$

So, 5.9×10^2 J of energy is lost to friction.

3- The change in kinetic energy of the box.

$\Delta E_k = E_k(B)-E_k(A)=W\sum(\vec{F})=W(\vec{F}_{app})+W(\vec{F}_f)$

$\vec{N}$ and $m\vec{g}$ do no work on the box as it moves, because they are perpendicular to the deplacement.

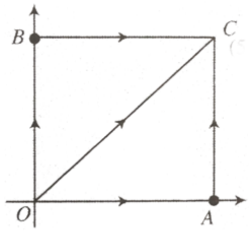
$\Delta E_k = W(\vec{F}_{app})+W(\vec{F}_f)=650-590 \Rightarrow \mathbf{\Delta E_k=60\ J}$

$$\mathbf{W_{A\to B}=E_k(B)-E_k(A)=\frac{mv_B^2}{2}-\frac{mv_A^2}{2}=\Delta E_k}$$

4- The final velocity of the box.

$$\mathbf{W=\Delta E_k=E_k(5\ m)-E_k(0\ m)=\frac{mv_f^2}{2}-0} \Rightarrow V_f=\sqrt{\frac{2\,\Delta E_k}{m}}=\sqrt{\frac{2\times 60}{40}} \Rightarrow V_f=1.73\ m/s$$

EXERCISE 04



$F_f=3$ N.

The work done by the friction force.

(a) path OA and return path AO,

$$\mathbf{W_{A\to B}}(\vec{F}_f)=\int_A^B \vec{F}_f.\overrightarrow{dr}=\int_A^B F_f.\,dr.\cos\theta=-F_f.\,d.\cos 180=-F_f.\,d$$

$$\mathbf{W_{O\overset{1}{\to}O}(\vec{F}_f)=W_{O\to A}(\vec{F}_f)+W_{A\to O}(\vec{F}_f)}=-F_f.OA+(-F_f.AO)=-3\times 5-3\times 5$$

$$\Rightarrow \mathbf{W_{O\overset{1}{\to}O}(\vec{F}_f)=-30\ J}$$

(b) path OA followed by AC and the return path CO.

$$\mathbf{W_{O\overset{2}{\to}O}(\vec{F}_f)=W_{O\to A}(\vec{F}_f)+W_{A\to C}(\vec{F}_f)+W_{C\to O}(\vec{F}_f)}=-F_f.OA+(-F_f.AC)+(-F_f.CO)$$

$$W_{O\overset{2}{\to}O}(\vec{F}_f)=-3\times 5-3\times 5-3\times\sqrt{50}\Rightarrow \mathbf{W_{O\overset{2}{\to}O}(\vec{F}_f)=-51.2\ J}$$

(c) path OC followed by the return path CO.

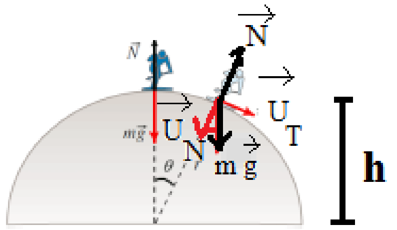
$$\mathbf{W_{O\overset{3}{\to}O}(\vec{F}_f)=W_{O\to C}(\vec{F}_f)+W_{C\to O}(\vec{F}_f)}=-F_f.OC+(-F_f.CO)=-3\times\sqrt{50}-3\times\sqrt{50}$$

$$\Rightarrow \mathbf{W_{O\overset{3}{\to}O}(\vec{F}_f)=-42.4\ J}$$

It can be observed that: $\mathbf{W_{O\overset{1}{\to}O}\neq W_{O\overset{2}{\to}O}\neq W_{O\overset{3}{\to}O}}$. So, we can conclude that the friction force is non-conservative force.

EXERCISE 05

A skier of mass *m* starts sliding from rest at the top of a solid frictionless hemisphere of radius *r*. At what angle θ will the skier leave the sphere?



By the application of FPD:

$$\sum \overrightarrow{F_{ext}} = m\,\vec{a}$$

$\vec{W} + \vec{N} = m\vec{a} \Rightarrow$

$$\begin{cases} mg\sin\theta = ma_T \\ mg\cos\theta - N = m\frac{V^2}{R} \Rightarrow N = mg\cos\theta - m\frac{V^2}{R} \end{cases}$$

When the skiers leaves the sphere, N= 0 $\Rightarrow N = mg\cos\theta - m\frac{V^2}{R} = 0 \Rightarrow mg\cos\theta = m\frac{V^2}{R}$

$$\Rightarrow V^2 = Rg\cos\theta = Rg\frac{h}{R} \Rightarrow V^2 = g\,h$$

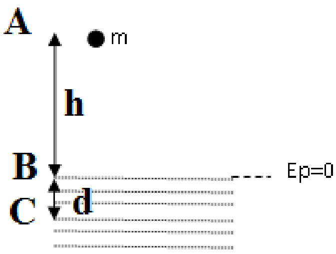
The motion is without friction $\Rightarrow \Delta E_M = 0 \Rightarrow E_{M2} = E_{M1} \Rightarrow E_{k2} + E_{p2} = E_{k1} + E_{p1}$

$$\Rightarrow \frac{mV^2}{2} + mgh = 0 + mgR \Rightarrow \frac{gh}{2} + gh = gR \Rightarrow \frac{h}{2} + h = R \Rightarrow \frac{h}{2} + h = R$$

$$\Rightarrow h = \frac{2}{3}R$$

EXERCISE 06

m=5 g, h=14.8 m, V_A=10 m/s. d =20 cm. g = 10 m/s^2.



1- Calculation of the velocity of the ball when it reaches the surface of the sand.

The air resistance is neglected $\Rightarrow \Delta E_M = 0 \Rightarrow E_M(B) = E_M(A)$

$$\Rightarrow E_k(B) + E_p(B) = E_k(A) + E_p(A)$$

$$\Rightarrow \frac{mV_B^2}{2} + 0 = \frac{mV_A^2}{2} + mgh$$

$$\Rightarrow V_B = \sqrt{V_A^2 + 2gh} = \sqrt{10^2 + 2 \times 10 \times 14.8} \Rightarrow V_B = 19.9\ m/s$$

2- The magnitude of the average force exerted by the sand on the ball.

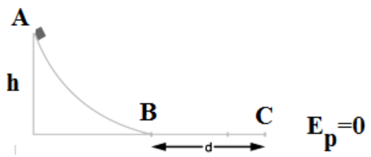
$$\Rightarrow \Delta E_M = W_{\vec{F_f}}$$

$$\Rightarrow (E_{c2} + E_{p2}) - (E_{c1} + E_{p1}) = W_{\vec{F_f}} \Rightarrow 0 - mgd - \left(\frac{mV_B^2}{2} + 0\right) = W_{\vec{F_f}}$$

$$\Rightarrow -mgd - \frac{mV_B^2}{2} = -F_f.\,d \Rightarrow F_f = mg + \frac{mV_B^2}{2d}$$

$$\Rightarrow F_f = 5 \times 10^{-3} \times 10 + \frac{5\times10^{-3}\times19.9^2}{2\times0.2} \Rightarrow F_f = 5\ N$$

EXERCISE 07



In the part AB, the motion is frictionless: the kinetic energy E_k available is equal to the loss of potential energy, ***mgh***:

$\Delta E_M = 0 \Rightarrow$ $\quad E_M(B) = E_M(A)$

$$\Rightarrow E_k(B) + E_p(B) = E_k(A) + E_p(A)$$

$$\Rightarrow \frac{mV_B^2}{2} + 0 = 0 + mgh$$

$$\Rightarrow E_k(B) = \frac{mV_B^2}{2} = mgh$$

On the flat track the entire kinetic energy is used up in the work done against friction

$$\Rightarrow \Delta E_M = W_{\vec{F_f}}$$

$$\Rightarrow (E_k(C) + E_p(C)) - (E_k(B) + E_p(B)) = W_{\vec{F_f}} \Rightarrow 0 - mgd - (\frac{mV_B^2}{2} + 0) = W_{\vec{F_f}}$$

$$\Rightarrow (0+0) - (\frac{mV_B^2}{2} + 0) = - F_f . d$$

$$\Rightarrow -(mgh) = -\mu \, mgd$$

$$\Rightarrow d = \frac{h}{\mu}$$

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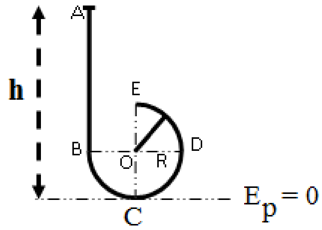
EXERCISE 08

A ball of mass m is released from a height H without initial velocity. AB is a vertical surface and BCDE is a ¾ of a circle of radius R.



1- The ball moves without friction:

The motion is without friction $\Rightarrow \mathbf{\Delta E_M = 0}$

a- The velocity of the ball at point B.

$\Delta E_M = 0 \Rightarrow E_M(B) = E_M(A) \Rightarrow E_k(B) + E_p(B) = E_k(A) + E_p(A)$

$$\Rightarrow \frac{mV_B^2}{2} + mgR = 0 + mgh \Rightarrow \mathbf{V_B} = \sqrt{\mathbf{2g(h - R)}}$$

b- The velocity of the ball at point C.

$\Delta E_M = 0 \Rightarrow E_M(C) = E_M(A) \Rightarrow E_k(C) + E_p(C) = E_k(A) + E_p(A)$

$$\Rightarrow \frac{mV_C^2}{2} + 0 = 0 + mgh \Rightarrow \mathbf{V_c} = \sqrt{\mathbf{2g\,h}}$$

c- The value of h for which the ball reaches the point E with a velocity $\sqrt{2gR}$.

$\Delta E_M = 0 \Rightarrow E_M(E) = E_M(A) \Rightarrow E_k(E) + E_p(E) = E_k(A) + E_p(A)$

$$\Rightarrow \frac{mV_E^2}{2} + mg(2R) = 0 + mgh \Rightarrow \frac{m2gR}{2} + mg(2R) = 0 + mgh \Rightarrow \mathbf{h = 3R}$$

2- the value of F_f if the ball just reaches point E (the velocity at point E is zero) and the motion takes place with a constant tangential friction force F_f in the BCDE part only.

$$\Delta E_M = W_{\vec{F_f}}$$

$$\Rightarrow (E_k(E) + E_p(E)) - (E_k(B) + E_p(B)) = W_{\vec{F_f}} \Rightarrow$$

$$0 + 2mgR - \left(\frac{mV_B{}^2}{2} + mgR\right) = W_{\overrightarrow{F_f}}$$

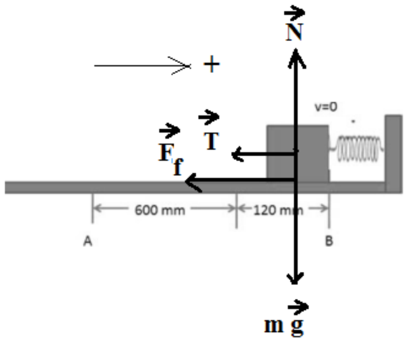
$$W_{\overrightarrow{F_f}} = \int_A^B \overrightarrow{F_f} . \overrightarrow{dr}$$

$$\overrightarrow{F_f} = -F_f \overrightarrow{U_T} \;,\; \overrightarrow{dr} = dr\,\overrightarrow{U_T} = Rd\theta\,\overrightarrow{U_T}$$

$$W_{\overrightarrow{F_f}} = -F_f . R \int_0^{\frac{3\pi}{2}} d\theta \Rightarrow W_{\overrightarrow{F_f}} = -\frac{3\pi}{2} R . F_f$$

$$\Rightarrow mgR - \frac{\mathbf{m \times 2g(h-R)}}{2} = -\frac{3\pi}{2} R . F_f \Rightarrow \mathbf{F_f} = \frac{\boldsymbol{2mg}}{\boldsymbol{3\pi R}} \left(\boldsymbol{2R - h}\right)$$

EXERCISE 09



a- The work done by the spring W_S.

$$W_S = W(\vec{T}) = \int_C^B \vec{T} . \overrightarrow{dx} = -\int_A^B T . dx = -\int_A^B Kx . dx \Rightarrow W_S = -\frac{1}{2} kx^2 = -\frac{1}{2}\, 20 \times 10^3 \times (0.12)^2$$

$$\Rightarrow W_S = -144 \text{ J}$$

b- The work done by the friction force W_{F_f}.

$$\Delta \mathbf{E_k} = \mathbf{E_k(B)} - \mathbf{E_k(A)} = \mathbf{W} \sum(\vec{F}) = W_S + W_{F_f}$$

$\vec{N}$ and $m\vec{g}$ do no work on the box as it moves, because they are perpendicular to the deplacement.

$$\Delta \mathbf{E_k} = \frac{mV_B{}^2}{2} - \frac{mV_A{}^2}{2} = W_S + W_{F_f}$$

$$\Rightarrow \Delta \mathbf{E_k} = 0 - \frac{mV_A{}^2}{2} = W_S + W_{F_f} \Rightarrow W_{F_f} = -\frac{mV_A{}^2}{2} - W_S$$

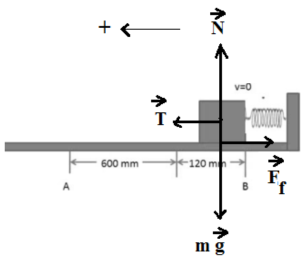
$$\Rightarrow W_{F_f} = -\frac{50 \times 3^2}{2} - (-144) \Rightarrow \mathbf{W_{F_f}} = \mathbf{-81}\, \boldsymbol{J}$$

c- The coefficient of friction between the crate and the surface.

$$\mathbf{W_{F_f}} = -F_f . d = -\mu . N . d = -\mu .mg.d = -81 \text{ J}$$

$$\Rightarrow \mu = \frac{81}{50 \times 9.8 \times (0.6 + 0.12)} \Rightarrow \mu = \mathbf{0.2296}$$

d- The velocity of the crate as it passes again through position A after rebounding off the spring.



$$\Delta \mathbf{E_k} = \frac{mV_A{}^2}{2} - \frac{mV_B{}^2}{2} = W_S + W_{F_f}$$

$$\Rightarrow \mathbf{E_k} = \frac{mV_A{}^2}{2} - 0 = W_S + W_{F_f}$$

$$\Rightarrow V_A = \sqrt{\frac{2}{m}(W_S + W_{F_f})}$$

$$W_S = W(\vec{T}) = \int_B^C \vec{T}.\overrightarrow{dx} = \int_B^C T.\,dx = \int_B^C Kx.\,dx \Rightarrow W_S = \frac{1}{2}kx^2 = \frac{1}{2}\,20 \times 10^3 \times (0.12)^2$$

$$\Rightarrow \mathbf{W_S = 144\ J}$$

$$\mathbf{W_{F_f}} = -F_f.d = -\mu.N.d = -\mu.mg.d = -81\ J$$

$$\Rightarrow V_A = \sqrt{\frac{2}{50}(144 - 81)}$$

$$\Rightarrow \mathbf{V_A = 1.587\ m/s}$$