Mohamed Boudiaf University of Msila.
Faculty of sciences
Field : Sciences of matter (SM)
1st year LMD   Semester 01.

جامعة محمد بوضياف -المسيلة
كلية العلوم
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Physics 01: Mechanics of point particle.

University Year 2023-2024

Correction of the evaluation exam

EXERCISE 01: (6 pts)

Let's be: $\vec{A} = -5\vec{i} - 3\vec{j} + 2\vec{k}$ and $\vec{B} = -2\vec{j} - 2\vec{k}$

1- Calculation of the magnitude for each vector.

$\|\vec{A}\| = \sqrt{38}$, $\|\vec{B}\| = 2\sqrt{2}$ **1**

2- Calculation of $\vec{A}.\vec{B}$ *and* $\vec{A} \wedge \vec{B}$.

$\vec{A}.\vec{B} = (-5).(0) + (-3).(-2) + (2).(-2) \Rightarrow \vec{A}.\vec{B} = 2$ **1**

$$\vec{A} \wedge \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -3 & 2 \\ 0 & -2 & -2 \end{vmatrix} = 10\,\vec{i} - 10\,\vec{j} + 10\,\vec{k}$$ **1**

3- Calculation of the angle between $\vec{A}$ and $\vec{B}$.

$$\vec{A}.\vec{B} = A.B.\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{A.B} = \frac{2}{\sqrt{38}.\,2\sqrt{2}} = \frac{1}{\sqrt{38}.\,\sqrt{2}} \Rightarrow \cos\theta = 0.115 \Rightarrow \theta = 83.4^{o}$$ **1**

4- The components of a vector $\vec{C}$ that is perpendicular to $\vec{B}$, is in the (yoz) plane and has a magnitude of 5 units.

- $\vec{C}$ is in the (yoz) $\Rightarrow \vec{C} = y_C\vec{j} + z_C\vec{k}$
- $\vec{C}$ is perpendicular to $\vec{B} \Rightarrow \vec{C}.\vec{B} = 0 \Rightarrow -2y_C - 2z_C = 0 \Rightarrow y_C = -z_C$
- $\vec{C}$ has a magnitude of 5 units $\Rightarrow \|\vec{C}\| = \sqrt{y_C^2 + z_C^2} = 5$

  $\Rightarrow y_C = -\frac{5}{\sqrt{2}},\ z_C = \frac{5}{\sqrt{2}}$ or $y_C = \frac{5}{\sqrt{2}},\ z_C = -\frac{5}{\sqrt{2}}$ **2**

EXERCISE 02: (8pts)

1- $y = At^2 - Bt^3$

The dimensions of A and B:

$$[L] = [A].[T]^2 - [B].[T]^3$$

$\Rightarrow [A] = [L].[T]^{-2}$, $[B] = [L].[T]^{-3}$ **1**

2- The various coordinates systems, the coordinates and unit vectors of each referential. **3**

Reference frame	Coordinates	Unit vectors
Cartesian coordinates system	(x, y, z)	$(\vec{i}, \vec{j}, \vec{k})$
Polar coordinates system	(ρ, θ)	$(\vec{u}_\rho, \vec{u}_\theta)$
Cylindrical coordinates system	(ρ, θ, z)	$(\vec{u}_\rho, \vec{u}_\theta, \vec{k})$
Spherical coordinates system	(r, θ, Φ)	$(\vec{u}_r, \vec{u}_\theta, \vec{u}_\Phi)$

3-a- Definition of the average speed, and the average velocity.

The average velocity is a vector quantity and is defined as follow: **1**

$$\vec{V}_{avg} = \frac{\overrightarrow{OM_2} - \overrightarrow{OM_1}}{t_2 - t_1} = \frac{\overrightarrow{M_1M_2}}{t_2 - t_1} = \frac{\Delta\overrightarrow{OM}}{\Delta t}$$

The average speed is positive scalar quantity, it's the distance traveled per unit time: **1**

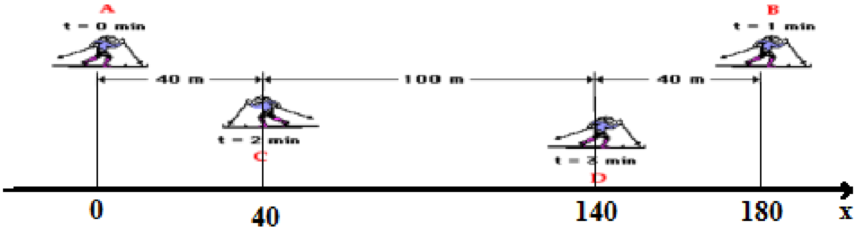
$$S_{avg} = \frac{traveled\ distance}{\Delta t}$$

3-b- Calculation of the average speed and the average velocity of the skier during:

- 0 min and 3 min.

If the motion is unidirectional (one direction), for example in the direction of the axis (OX), the velocity can be expressed as follow:

$$V_{avg} = \frac{x_3 - x_0}{t_3 - t_0} = \frac{\Delta x}{\Delta t}$$



$$V_{avg}(0 - 3\,min) = \frac{140-0}{(3-0)\times 60} = 0.77 \text{ m/s} \qquad \underline{0.5}$$

$$V_{avg}(0 - 3\,min) = \frac{180+140+100}{(3-0)\times 60} = 2.33 \text{ m/s} \qquad \underline{0.5}$$

- 1 min and 3 min.

  - $V_{avg}(1 - 3\,min) = \frac{140-180}{(3-1)\times 60} = -\,0.33 \text{ m/s}$ **0.5**
  - $V_{avg}(1 - 3\,min) = \frac{140+100}{(3-1)\times 60} = 2 \text{ m/s}$ **0.5**

EXERCISE 03: (7 pts)

The polar coordinates of a material point are : $\rho(t) = ae^{\theta}$, $\theta = wt$, w: constant, a: constant

1- The vector position in polar coordinates.

$$\overrightarrow{OM} = \rho\vec{u}_\rho \Rightarrow \overrightarrow{OM} = ae^{wt}\,\vec{u}_\rho \qquad \underline{0.5}$$

2- The velocity and acceleration in polar coordinates and their magnitudes.

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \frac{d(ae^{wt}\,\vec{u}_\rho)}{dt} = awe^{wt}\vec{u}_\rho + awe^{wt}\vec{u}_\theta$$

$$\vec{V} = awe^{wt}(\vec{u}_\rho + \vec{u}_\theta) \qquad \underline{1}$$

$$V = \sqrt{2}\,awe^{wt} \qquad \underline{0.5}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = aw^2e^{wt}(\vec{u}_\rho + \vec{u}_\theta) + awe^{wt}(w\,\vec{u}_\theta - w\,\vec{u}_\rho)$$

$$\vec{a} = 2\,aw^2e^{wt}\,\vec{u}_\theta \qquad \underline{1}$$

$$a = 2\,aw^2e^{wt} \qquad \underline{0.5}$$

2- Calculate the tangential acceleration and the normal acceleration.

$$\begin{cases} a_T = \frac{dV}{dt} \\ a_N = \frac{V^2}{\Re} = \sqrt{a^2 - a_T^2} \end{cases} \Rightarrow \begin{cases} a_T = \sqrt{2}\,aw^2e^{wt} \\ a_N = \sqrt{(2\,aw^2e^{wt})^2 - (\sqrt{2}\,aw^2e^{wt})^2} = \sqrt{2}\,aw^2e^{wt} \end{cases} \qquad \underline{2}$$

4- Deduce the radius of curvature.

$$a_N = \frac{V^2}{\Re} \Rightarrow \Re = \frac{V^2}{a_N} \Rightarrow \Re = \frac{(awe^{wt}\sqrt{2})^2}{\sqrt{2}\,aw^2e^{wt}} \Rightarrow \Re = \sqrt{2}\,ae^{wt} \qquad \underline{0.5}$$

5- Calculate the curvilinear abscissa S(t) as a function of time.

$$V = \frac{dS}{dt} \Rightarrow dS = V.dt \Rightarrow \int dS = \sqrt{2}\,a\int we^{wt}dt \Rightarrow S = \sqrt{2}\,ae^{wt} + C \qquad \underline{1}$$