

Practice Sheets N°01 (Analysis 2)

Exercise 01

Find the antiderivatives or evaluate the definite integral in each problem

$$\int_0^1 (x^2 + \sqrt{x}) dx; \quad \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx; \quad \int \frac{1}{x \ln x} dx; \quad \int_0^{\pi} \sin(nx) dx$$
$$\int \frac{1 - \cos x}{x - \sin x} dx; \quad \int \frac{xe^x + e^x}{xe^x + 1} dx; \quad \int \frac{\tan(x)}{\cos^2(x)} dx; \quad \int_0^4 |x - 2| dx$$
$$\int \frac{\sin(x)}{\cos^2(x)} dx; \quad \int \frac{x}{(x^2 + 1)^3} dx; \quad \int \tan(x) dx; \quad \int e^{3x+2} dx;$$
$$\int \cos(x) \sin^3(x) dx; \quad \int 2x(x^2 + 3)^5 dx;$$

Exercise 02

Find the indicated integrals

$$\int \frac{\cos(\ln(x))}{x} dx; \quad \int \frac{1}{x\sqrt{1-\ln^2(x)}} dx; \quad \int \frac{e^x}{e^{2x} + 1} dx$$

Exercise 03

Find the following integrals

$$\int xe^{-x} dx; \quad \int x \ln(x) dx; \quad \int \arctan(x)$$
$$\int_0^{\pi} x \sin(nx) dx;$$

Exercise 04

Find the antiderivatives or evaluate the definite integral in each problem

$$\int \frac{3x - 5}{(x - 1)(x + 1)} dx; \quad \int \frac{3x + 2}{x^2 - 3x + 2} dx; \quad \int \frac{3x + 2}{(x - 1)^2(x + 5)} dx; \quad \int \frac{x + 2}{(2x + 1)(x^2 + 1)} dx;$$
$$\int \frac{1}{x^2 - 4} dx; \quad \int \frac{3}{(x^2 - 1)(x^2 - 4)} dx; \quad \int \frac{x + 2}{x^2 - 6x + 2} dx;$$
$$\int \frac{1}{x^2 + 5} dx; \quad \int \frac{x}{x^2 + x + 1} dx; \quad \int \frac{x^2}{(x^2 - 3x + 2)} dx;$$

Exercise 05

Find the integral in each of the following cases

$$\int \frac{e^x}{e^{2x} - 3e^x + 2} dx; \quad \int (x^2 - 3x + 1)e^x dx; \quad \int \frac{1}{\operatorname{ch}(x)} dx; \quad \int \frac{e^x}{e^{2x} - 1} dx$$

Exercise 06

Find the integral in each of the following cases

$$\int \sin^3(x) \cos^2(x) dx; \quad \int \sin^2(x) \cos^3(x) dx; \quad \int \sin(2x) \cos(3x) dx$$

$$\int \frac{1}{\sin(x)} dx;$$

Exercise 07

$$\int \frac{5}{3+x^2} dx; \quad \int \frac{\ln^5(x)}{x} dx; \quad \int \frac{1+\ln(x)}{1+x \ln(x)} dx; \quad \int x^2 e^{2x} dx$$

$$\int x \sin(x) dx; \quad \int \frac{\sin(x) \cos(x) dx}{\sin(x)+1}; \quad \int \frac{\sin(x)}{1+\cos(x)} dx; \quad \int \frac{1}{\cos(x)} dx;$$

$$\int \sin^2(x) \cos^2(x) dx; \quad \int \cos(x) e^x dx; \quad \int x \ln(x^2) dx; \quad \int x e^{3x^2} dx$$

Exercise 08

Let

$$I_n = \int_0^1 \frac{x^n}{1+x} dx, n \in \mathbb{N}$$

1. Evaluate I_0
2. Show that $I_n + I_{n+1} = \frac{1}{n+1}$
3. Deduce I_1 and I_2

Exercise 09

$$I = \int e^{2x} \cos^2(x) dx \text{ and } J = \int e^{2x} \sin^2(x) dx$$

1. Evaluate $I + J$
2. Evaluate $I - J$
3. Deduce I and J

Exercise 10

1. It is estimated that t years from now the population of a certain lakeside community will be changing at the rate of $0.6t^2 + 0.2t + 0.5$ thousand people per year. Environmentalists have found that the level of pollution in the lake increases at the rate of approximately 5 units per 1000 people. By how much will the pollution in the lake increase during the next 2 years?
2. An object is moving so that its speed after t minutes is $v(t) = 1 + 4t + 3t^2$ meters per minute. How far does the object travel during 3^{rd} minute?