Ministry of Higher Education and Scientific Research

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Mechanics of a Particle

Problems & Exercises (of Chap 0)

Vectors, Coordinate systems

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PART 1: Vectors

Exercise 1

1- Find magnitude and direction of a vector, $\mathbf{A} = (6\,\vec{i} - 8\,\vec{j})$
2- Obtain the magnitude of **2A - 3B;** if $\mathbf{A} = \vec{i} + \vec{j} - 2\,\vec{k}$; $\mathbf{B} = 2\vec{i} - \vec{j} + \vec{k}$
3- Resolve a weight of **10** ***N*** in two directions which are parallel and perpendicular to a slope inclined at **30°** to the horizontal.
4- Resolve horizontally and vertically a force $F = 8\ N$ which makes an angle of ***45°*** with the horizontal.

Exercise 2

In a system of orthonormal axes, the following vectors are given:

$$\vec{V}_1 = 3\vec{i} + 4\vec{j} \text{ and } \vec{V}_2 = -\vec{i} + 2\vec{j}.$$

1- Calculate the magnitudes of the vectors $\|\vec{V}_1\|$, $\|\vec{V}_2\|$, and $\|\vec{V}_1 + \vec{V}_2\|$.
2- Determine the angle θ between the vectors $\vec{V}_1$ and $\vec{V}_2$.
3- Determine the unit vector $\vec{u}$ carried by the vector $(\vec{V}_1 + \vec{V}_2)$.
4- Determine the angles α, β and γ that $\vec{u}$ makes with the coordinate axes (*ox*), (*oy*), and (*oz*), respectively.
5- Calculate the components of the vector $\vec{V}_3 = \vec{V}_1 \wedge \vec{V}_2$.
6- Consider the triangle (OAB) defined by the endpoints of vectors $\vec{V}_1$ and $\vec{V}_2$: $\vec{V}_1 = \overrightarrow{OA}$ and $\vec{V}_2 = \overrightarrow{OB}$. Show that the area of the triangle (OAB) can be expressed in terms of $\|\vec{V}_1\|$, $\|\vec{V}_2\|$, and θ.
Verify that it can also be obtained using the relation: $\frac{1}{2}\|\vec{V}_1 \wedge \vec{V}_2\|$.

Exercise 3

In the three-dimensional orthonormal coordinate system $(O, \vec{i}, \vec{j}, \vec{k})$, we have three vectors:

$$\vec{U} = \vec{i} + \vec{j} + \vec{k};\ \vec{V} = 2\vec{i} - \vec{j} + 2\vec{k};\ \vec{W} = -2\vec{k}$$

1. Draw the three vectors $\vec{U}$, $\vec{V}$, and $\vec{W}$.
2. Calculate the magnitudes of $\|\vec{U}\|$, $\|\vec{V}\|$, and $\|\vec{W}\|$.
3. Determine the components of the unit vector $\vec{u}$ carried by $\vec{U}$.
4. Graphically represent the vector $(\vec{U} - \vec{V})$ and calculate its magnitude.
5. Calculate:

a) The dot product $\vec{U} \cdot \vec{V}$.

b) The cross product $\vec{U} \wedge \vec{V}$.

c) The double cross product $(\vec{U} \wedge \vec{V}) \wedge \vec{W}$.

d) The scalar triple product $(\vec{V} \wedge \vec{W}) \cdot \vec{U}$.

6. Determine the angle between $\vec{U}$ and $\vec{V}$.

<u>Exercise 4</u>

Show that the magnitudes of the sum and the difference of two vectors $\vec{\boldsymbol{A}} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$ and $\vec{\boldsymbol{B}} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$, expressed in rectangular coordinates, are respectively:

$$\boldsymbol{S = [(A_x + B_x)^2 + (A_y + B_y)^2 + (A_z + B_z)^2]^{1/2}}$$

$$\boldsymbol{S = [(A_x - B_x)^2 + (A_y - B_y)^2 + (A_z - B_z)^2]^{1/2}}$$

<u>Exercise 5</u>

Show that the area of a parallelogram is $|\vec{A} \wedge \vec{B}|$, where $|\vec{A}|$ and $|\vec{B}|$ are the sides of the parallelogram formed by the two vectors.

b/ Prove that vectors $\vec{A}$ and $\vec{B}$ are perpendicular if: $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$.

<u>Exercise 6</u>

Let's consider the two vectors $\vec{A}\begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix}$ and $\vec{B}\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$. Find $\alpha; \beta$ such that $\vec{B}$ is parallel to $\vec{A}$, then determine the unit vector for each of the two vectors.

<u>Exercise 7</u>

Consider the scalar field $\boldsymbol{(x, y, z) = 3x^2y + y^2z^2}$ and the vector field given by:

$$\vec{\boldsymbol{V}}\,\boldsymbol{(x, y, z) = xz^2\vec{\imath} + (2x^2 - y)\,\vec{\jmath} + yz^2\vec{k}}$$

Calculate: the gradient ∇f, the divergence div($\vec{V}$), the curl rot($\vec{V}$).

PART 2: Coordinate System

Exercise 8

1- Convert the following vector from Cartesian coordinates $(\vec{i}, \vec{j})$ to polar coordinates $(\vec{u}_\rho, \vec{u}_\theta)$

$$\vec{V} = X\,\vec{i} + Y\,\vec{j}$$

2- Convert the following vector from Cartesian coordinates $(\vec{i}, \vec{j}, \vec{k})$, to cylindrical coordinates $(\vec{u}_\rho, \vec{u}_\theta, \vec{u}_Z)$.

$$\vec{V} = X\,\vec{i} + Y\,\vec{j} + Z\,\vec{k}$$

Exercise 9

Convert the equation written in Spherical coordinates into an equation in Cartesian coordinates.

$$\csc\theta = 2\cos\varphi + 4\sin\varphi$$

Exercise 10

Convert the cylindrical coordinates **($\rho = 5$, $\theta = 2\pi/3$, $z = -7$)** into Cartesian coordinates ***(x, y, z).***