



PART 1: Characteristics of Motion

Exercise 1

The position of the material point M is defined in a direct orthonormal coordinate system $\mathcal{R}(O, \vec{i}, \vec{j}, \vec{k})$, with x and y in centimeters and t in seconds.

$$x(t) = 2t - 2$$

$$y(t) = t^2 - 2t + 3$$

1. Write the equation for the motion's trajectory, $y = f(x)$, and determine its nature.
2. Provide the expression of the position vector \overrightarrow{OM} .
3. Determine the components of the velocity vector \vec{v} and deduce its magnitude $|\mathbf{v}|$.
4. Determine the components of the acceleration vector \vec{a} and its magnitude $|\mathbf{a}|$.
5. Calculate the radius of curvature $\rho = v^3 / |\vec{v} \wedge \vec{a}|$ at the instant $t = 0$.

Exercise 2

In a plane (P) with an orthonormal coordinate system xOy , there is a mobile point M moving in this plane. At time t , its coordinates are defined as follows:

$$x = \sqrt{2} \cos(t/2)$$

$$y = 2\sqrt{2} \sin(t/2)$$

a/ What is the trajectory?

b/ Calculate the coordinates at time t of the velocity vector \vec{v} and the acceleration vector \vec{a} of this mobile point.

What is the relationship between \overrightarrow{OM} and \vec{a} ? How long does it take the mobile point to return to the same position on the curve?

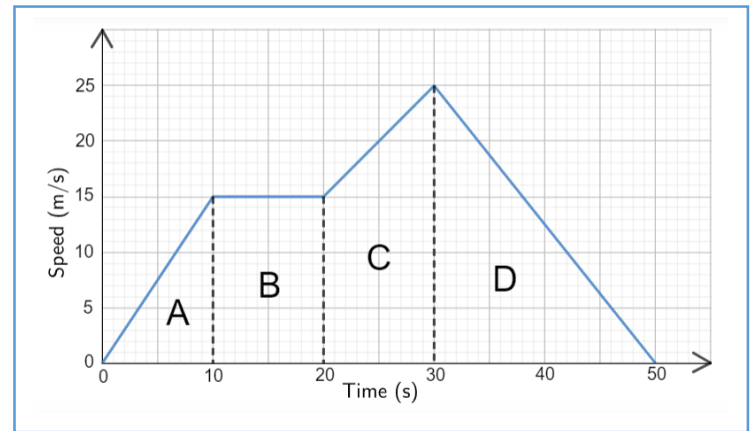
c/ Between the times $t_1 = 0$ and $t_2 = 4\pi$, determine the positions of the mobile point and the coordinates of \vec{v} to have an acceleration vector of length $\sqrt{5}/4$.

PART 2: Rectilinear Motion

Exercise 3

The speed-time graph shows a 50-second car journey.

- 1/ Describe the 50 second journey.
- 2/ Find which section of the graph has the greatest acceleration.
- 3/ Calculate the total distance travelled over the 50 seconds.



Exercise 4

A man at the top of a building vertically throws a ball upward with a velocity of **12 m/s**. The ball reaches the ground **4.25 seconds** later. ($g = 9.8 \text{ m/s}^2$)

- 1/ What is the maximum height reached by the ball?
- 2/ What is the height of the building?
- 3/ With what velocity does it reach the ground?

PART 3: Motion in the Plane

Exercise 5

The flat trajectory of a material point in polar coordinates is given by the equation:

$$\rho \cos^2 \frac{\theta}{2} = a, \text{ where } a \text{ is a constant.}$$

It is assumed that the magnitude v of the velocity of this material point is proportional to ρ :

$$v = k \rho, \text{ where } k \text{ is a positive constant.}$$

Calculate the normal v_ρ and transverse v_θ components of the velocity vector.

PART 5: Motion in the Space

Exercise 6

Consider a mobile point M in motion such that:

$$\overrightarrow{OM} = 3 \cos 2t \vec{i} + 3 \sin 2t \vec{j} + (8t - 4) \vec{k}$$

- 1/ Determine the nature of the trajectory of M in space (O, x, y, z).
- 2/ Provide, in cylindrical coordinates, the expression for the velocity. Calculate its magnitude.
- 3/ Provide, in cylindrical coordinates, the expression for the acceleration. Calculate its magnitude.