Physics 1st Exam Solution

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Exercise 1: (06 points)

1) ***The time equation of the uniform rectilinear motion***

We choose the ***OX*** axis as a rectilinear reference frame and establish the initial condition ***t = 0; x = x_0*** (initial abscissa).

Starting from the definition above and through integration, we can express the abscissa *x* as a function of time.

0.5 $$v = \dot{x} = \frac{dx}{dt} = v_0 \Rightarrow dx = v_0.dt \Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} v_0.dt$$ 0.5

0.5

$$x\Big|_{x_0}^{x} = v_0 t\Big|_0^t \quad \Rightarrow x\text{-}x_0 = v_0 t \qquad \boldsymbol{x = v_0\, t + x_0}$$ 0.5

2) ***Velocity and Acceleration in Polar Coordinates.***

We can express the expressions of the two unit vectors $\overrightarrow{u_\rho}$ and $\vec{u}_\theta$ in terms of the unit vectors $\vec{\imath}$ and $\vec{\jmath}$:

$$\begin{cases} \overrightarrow{u_\rho} = \cos\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath} \\ \overrightarrow{u_\theta} = -\sin\theta\,\vec{\imath} + \cos\theta\,\vec{\jmath} \end{cases}$$

Their consecutive derivatives are:

$$\frac{\overrightarrow{du_\rho}}{dt} = -\sin\theta\frac{d\theta}{dt}\vec{\imath} + \cos\theta\frac{d\theta}{dt}\vec{\jmath} = \vec{u}_\theta\frac{d\theta}{dt}$$

$$\frac{\overrightarrow{du_\theta}}{dt} = -\cos\theta\frac{d\theta}{dt}\vec{\imath} - \sin\theta\frac{d\theta}{dt}\vec{\jmath} = -\vec{u}_\rho\frac{d\theta}{dt}$$

$$\boxed{\frac{\overrightarrow{du_\rho}}{dt} = \vec{u}_\theta\frac{d\theta}{dt}}$$ 0.5 $$\boxed{\frac{\overrightarrow{du_\theta}}{dt} = -\vec{u}_\rho\frac{d\theta}{dt}}$$ 0.5

Using these relations, let's express velocity in polar coordinates:

$$\vec{v} = \dot{\vec{\rho}} = \rho\frac{\overrightarrow{du_\rho}}{dt} + \vec{u}_\rho\frac{d\rho}{dt} \Rightarrow \vec{v} = \frac{d\rho}{dt}\vec{u}_\rho + \rho\frac{d\theta}{dt}\vec{u}_\theta \Rightarrow \boxed{\vec{v} = \dot{\rho}\,\vec{u}_\rho + \rho\,\dot{\theta}\,\vec{u}_\theta}$$ 0.5 0.5

We differentiate the velocity relation, with respect to time, we obtain the formula for acceleration.

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{\rho}\,\overrightarrow{u_\rho} + \rho\dot{\theta}\overrightarrow{u_\theta})$$ 0.5 $$\Rightarrow \vec{a} = \dot{\vec{v}} = \dot{\rho}\frac{\overrightarrow{du_\rho}}{dt} + \ddot{\rho}\,\vec{u}_\rho + \rho\,\dot{\theta}\frac{\overrightarrow{du_\theta}}{dt} + \rho\,\ddot{\theta}\,\vec{u}_\theta + \dot{\rho}\,\dot{\theta}\,\vec{u}_\theta$$ 0.5

$$\vec{a}(t) = \ddot{\rho}\overrightarrow{u_\rho} + \dot{\rho}\dot{\theta}\overrightarrow{u_\theta} + \dot{\rho}\dot{\theta}\overrightarrow{u_\theta} + \rho\ddot{\theta}\overrightarrow{u_\theta} + \rho\dot{\theta}(-\dot{\theta}\,\overrightarrow{u_\rho})$$ 0.5 $$\Rightarrow \vec{a}(t) = (\ddot{\rho} - \rho\dot{\theta}^2)\overrightarrow{u_\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\overrightarrow{u_\theta}$$ 0.5

Exercise 2: (07 points)

1. ***The components of the velocity and acceleration vectors in Cartesian coordinates:***

$$\vec{v}(t) = \frac{d\overrightarrow{OM}}{dt}: \begin{cases} v_x = \frac{d\,(R.\cos\theta)}{dt} = -R.\theta^{\cdot}.\sin\theta \\ v_y = \frac{d\,(R.\sin\theta)}{dt} = R.\theta^{\cdot}.\cos\theta \\ v_z = \frac{d(h\theta)}{dt} = h.\theta^{\cdot} \end{cases}$$ 01 ,

$$\vec{a}(t) = \frac{d\vec{v}}{dt}: \begin{cases} a_x = \frac{d\,(-R.\theta^{\cdot}.\sin\theta)}{dt} = -R.\theta^{\cdot\cdot}.\sin\theta - R.\theta^{\cdot 2}.\cos\theta \\ a_y = \frac{d\,(R.\theta^{\cdot}.\cos\theta)}{dt} = R.\theta^{\cdot\cdot}.\cos\theta - R.\theta^{\cdot 2}.\sin\theta \\ a_z = \frac{d(h.\theta^{\cdot})}{dt} = h.\theta^{\cdot\cdot} \end{cases}$$ 01

2. ***The components of the velocity and acceleration vectors in Cylindrical coordinates:***

$\vec{v}(t) = \dot{\rho}\,\vec{u_\rho} + \rho\dot{\theta}\vec{u_\theta} + \dot{z}\,\vec{k}$ **0.5** ⟹ $\vec{v}(t) = R\,\dot{\theta}\,\vec{u}_\theta + h\,\dot{\theta}\,\vec{k}$ **0.5**

$\vec{a}(t) = (\ddot{\rho} - \rho\dot{\theta}^2)\vec{u_\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\vec{u_\theta} + \ddot{z}\,\vec{k}$ **0.5** ⟹ $\vec{a}(t) = -R\,\dot{\theta}^2\,\vec{u}_\rho + R\,\ddot{\theta}\,\vec{u}_\theta + h\,\ddot{\theta}\,\vec{k}$ **0.5**

$v = \sqrt{\dot{\rho}^2 + (\rho\,\dot{\theta})^2 + \dot{z}^2}$ **0.5** ⟹ $v = \sqrt{(R\,\dot{\theta})^2 + (h\,\dot{\theta})^2}$ ⟹ $v = \dot{\theta}\,\sqrt{R^2 + h^2}$ **0.5**

3. The trajectory is a helix. Helical motion is a composition of circular motion in the ***(Oxy)*** plane and rectilinear uniform motion along the vertical axis ***Oz***. **01**

4. **The time at which the material point reaches a height *z = 5h***

We use the equation for ***z(t)***: $z(t) = h\,\theta(t)$

Given that $\theta(t) = \omega t$, we substitute: $z(t) = h\,\omega t$

Now set $z = 5h$ ⟹ $5h = h\,\omega t$ **0.5** ⟹ $t = \frac{5}{\omega}$ **0.5**

Exercise 3: (07 points)

1. ***the maximum value of the angle α:***

$\sum \vec{F} = \vec{0}$ ⟹ $\vec{P} + \vec{N} + \vec{f_s} = \vec{0}$

By projecting on OX: $P_x - f_s = 0$ **0.25** ⟹ $f_s = P_x$ ⟹ $f_s = m\,g\,\sin\alpha$ **0.5**

By projecting on OY: $N - P_Y = 0$ **0.25** ⟹ $N = P_Y$ ⟹ $N = m\,g\,\cos\alpha$ **0.5**

We know that: $\mu_s = \frac{f_s}{N}$ ⟹ $f_s = \mu_s\,.\,N$ ⟹ $f_s = \mu_s.\,m\,g\,\cos\alpha$ **0.5**

⟹ $m\,g\,\sin\alpha = \mu_s.\,m\,g\,\cos\alpha$ **0.5**

⟹ $\frac{\sin\alpha}{\cos\alpha} = \mu_s$ ⟹ $\tan\alpha = \mu_s = 0.4$

⟹ $\alpha \approx 21.8°$ **0.5**

2. ***The acceleration of the mass m:***

$\sum \vec{F} = m\,\vec{a}_1$ ⟹ $\vec{P} + \vec{N} + \vec{f_k} + \vec{F_1} = m\,\vec{a}_1$

By projecting on OX: $P_x + F_1 - f_k = m\,a_1$ **0.25** ⟹ $a_1 = \frac{P_x + F_1 - f_k}{m}$ **0.25**

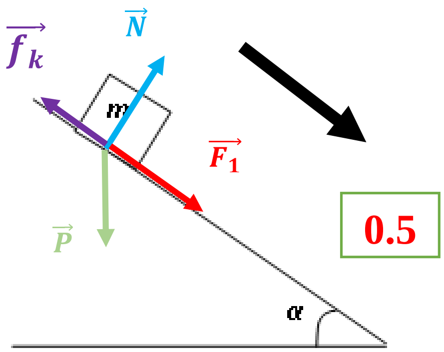
$a_1 = g\,(\sin\alpha - \mu_k.\cos\alpha) + \frac{F_1}{m}$ **01** ⟹ $a_1 \approx 1.85\ m/s^2$ **0.5**

3. The new acceleration of the mass m:

$$\sum \vec{F} = m\,\vec{a}_2 \Rightarrow \vec{P} + \vec{N} + \vec{f_k} + \vec{F_2} = m\,\vec{a}_2$$

By projecting on OX: $-P_x + F_2 - f_k = m\,a_2 \Rightarrow a_2 = \frac{-P_x + F_2 - f_k}{m}$

$a_2 = -g\,(\sin\alpha + \mu_k . \cos\alpha) + \frac{F_2}{m}$ **0.5** $\Rightarrow$ $\boldsymbol{a_2 \approx 1.18\ m/s^2}$ **0.5**



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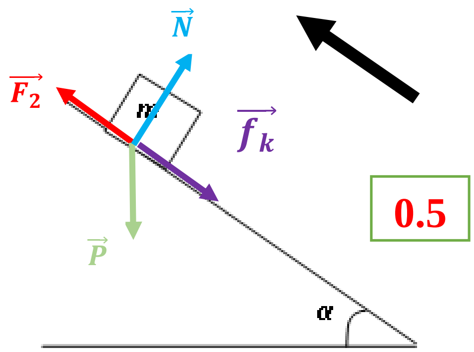

02