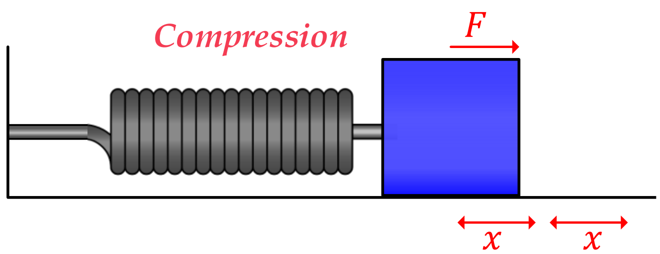
Experiment 04:

Simple Harmonic Motion in a Spring-Mass System



Experiment progression:

This report is prepared by:

Full name		Remarks	Group
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Simple Harmonic
Motion in a Spring-Mass System

1. Objectives

- Students will be able to **explain** the concepts of simple harmonic motion (SHM) in the context of a spring-mass system.
- Students will be able to **apply** their understanding of SHM and mechanical energy to **design** and conduct an experiment verifying the principle of energy conservation in the system.

2. Prior Knowledge

What a student should knew before the PW

- **Basic mechanics:** Concepts of force, motion (displacement, velocity, acceleration), Newton's Laws of Motion.
- **Work and Energy:** Understanding of work done by a force and different forms of energy (kinetic and potential).

3. Prior Knowledge Test

When an object accelerates due to a force, the object's acceleration is:

☐ Directly proportional to the force and inversely proportional to the mass.

☐ Directly proportional to both the force and the mass.

☐ Inversely proportional to the force and directly proportional to the mass.

☐ Independent of both force and mass.

4. **Definition:** A spring-mass system consists of a block attached to a free end of the spring. When it is displaced by x, the spring expands and then comes back

to its original position. In the absence of any resistance and friction, this spring oscillates infinitely. These motions are simple harmonic.

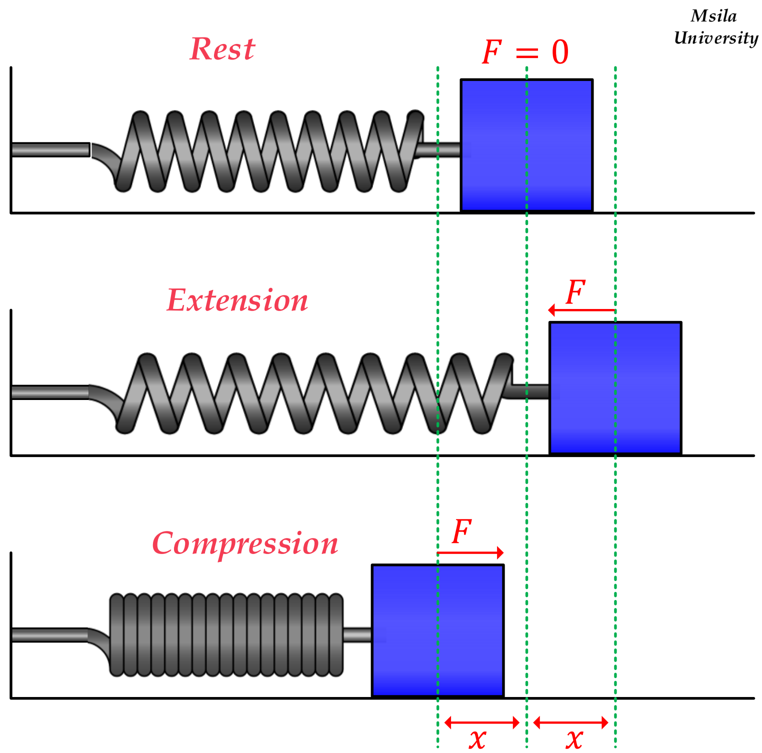

Fig. 1 A horizontal spring-mass system oscillating about the origin

5. System Description

We can describe the motion of the mass using energy principle, since the mechanical energy of the mass is conserved. At any position $x(t)$, the mechanical energy E_m, of the mass will have a term from the potential energy E_{ep} associated with the spring force, and kinetic energy E_C:

$$E_m = E_C + E_{ep}$$

Fig. 2 represents a graph of energy vs. displacement for a simple harmonic oscillator (a spring-mass system).

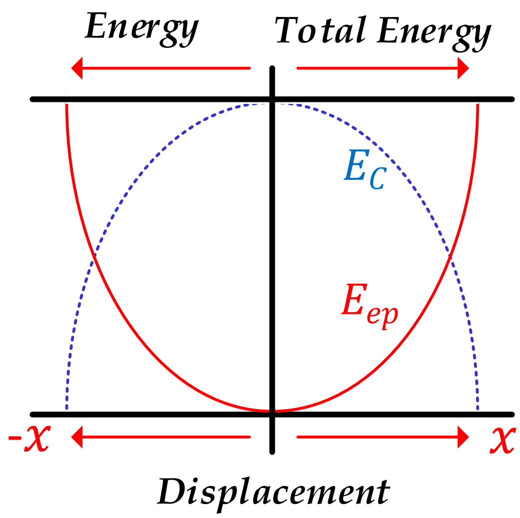

Fig. 2 Energy vs. displacement for a simple harmonic oscillator.

6. System Motion Equation

The motion of the system can be described with differential equation

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

where: $\omega_n = \sqrt{\frac{k}{m}}$ is the natural angular frequency of the system.

The general solution to this equation can be written in form of sinusoidal function

$$x(t) = X_m \cos(\omega_n t + \varphi)$$

7. Energy and simple harmonic oscillator

The kinetic energy of the block can be written as: $E_c = \frac{1}{2}mv^2 = \frac{mX_m^2\omega_n^2}{2}\sin^2(\omega_n t + \varphi)$

we have $\omega_n = \sqrt{\frac{k}{m}}$

So,

$$E_c = \frac{mX_m^2 k}{2m} \sin^2(\omega_n t + \varphi)$$

Thus

$$E_c = \frac{kX_m^2}{2} \sin^2(\omega_n t + \varphi)$$

Potential energy

$$E_{ep} = \frac{1}{2} kx^2 = \frac{kX_m^2}{2} \cos^2(\omega_n t + \varphi)$$

The mechanical energy is the sum of the kinetic and potential energies

$$E_m = E_C + E_{ep} = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

$$E_m = \frac{kX_m^2}{2} \sin^2(\omega_n t + \varphi) + \frac{kX_m^2}{2} \cos^2(\omega_n t + \varphi)$$

$$E_m = \frac{kX_m^2}{2} \underbrace{[\sin^2(\omega_n t + \varphi) + \cos^2(\omega_n t + \varphi)]}_{=1}$$

$$E_m = \frac{kX_m^2}{2}$$

As a conclusion, we can say that the mechanical energy of the system is constant and conserved

8. Demanded Work

1. Which of the following quantities remains constant in a simple harmonic motion of a mass-spring system?

☐ Acceleration

☐ Velocity

☐ Mechanical Energy (Total Energy)

☐ Displacement

2. Give the differential equation of the spring-mass system, then demonstrate that the $x(t) = X_m \sin(\omega_n t + \varphi)$ is the solution to the differential equation

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3. What is the velocity of the block when it first comes back to the equilibrium position?

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4. Demonstrate, using the differential equation, that mechanical energy is conserved

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5. Complete the table using the desktop graph simulator

Table: Energy Conservation Verification

	Masse m	**Spring constant** k	**Period** T	E_p	E_c	E_{ep}
Case 1	500 g	0.5 N/m	……	………..	…………	…………..
Case 2	750 g	1 N/m	……	………..	…………	……………
Case 3	1 kg	1.2 N/m	……	………..	…………	……………
Case 4	1.2 kg	1.5 N/m	……	………..	…………	…………..