Dynamics of the Material Point

INTRODUCTION:

Dynamics in physics is the science that studies the relationship between a moving body and the causes that bring about this motion. It also predicts the motion of a body situated within a specific environment. More precisely, dynamics is the analysis of the relationship between the applied force and the changes in the body's motion.

2/ NEWTON'S FIRST LAW 1642-1727 (GALILEAN PRINCIPLE OF INERTIA)

Statement of the principle:

If a material body is not subjected to any force, it is:

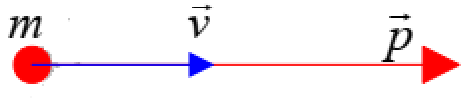
- Either in uniform rectilinear motion,
- Or at rest if it was initially at rest.

Statement: An isolated body (a body where the resultant of applied forces is zero, $\sum \vec{F} = \vec{0}$) remains at rest if it was initially at rest or maintains its uniform rectilinear motion ($\vec{a} = \vec{0}$) as long as the resultant of forces is zero. This holds true with respect to a frame or reference of inertia.

2/ MOMENTUM

Definition:

The momentum of a particle is the product of its mass by its instantaneous velocity vector.



$$\vec{P} = m\,\vec{v}$$

Momentum is a vector quantity. It's an essential concept because it introduces two elements that characterize the particle's state of motion: its mass and its velocity.

We can now provide a new statement of the principle of inertia: "**A free particle always moves with a constant momentum**."

Conservation of momentum:

If there is a variation in velocity or momentum, it implies that the particle is not free. Let's suppose the existence of two free particles subjected only to mutual influences between them; hence, they are isolated from the rest of the universe:

At time t: $\vec{p} = m_1.\vec{v}_1 + m_2.\vec{v}_2$

At time t': $\vec{p}\,' = m_1.\vec{v}_1^{\,'} + m_2.\vec{v}_2^{\,'}$

The experiments have shown that $\overrightarrow{\boldsymbol{P}} = \overrightarrow{\boldsymbol{P'}}$ constant, meaning that the total momentum of a system composed of two particles subjected only to their mutual influences remains constant.

In the case of two particles: $\overrightarrow{\boldsymbol{P_1}} + \overrightarrow{\boldsymbol{P_2}} = \boldsymbol{C^{te}}$

Between instants $\boldsymbol{t}$ and $\boldsymbol{t'}$: $p = p'$

$$\vec{p} = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$\vec{p}\,' = m_1 \vec{v}_1^{\,'} + m_2 \vec{v}_2^{\,'}$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1^{\,'} + \vec{p}_2^{\,'}$$

$$\vec{p}_1^{\,'} - \vec{p}_1 = \vec{p}_2 - \vec{p}_2^{\,'} \quad \Rightarrow \quad \Delta \vec{p}_1 = -\Delta \vec{p}_2$$

In the case of multiple particles

$$\left.\begin{aligned} \vec{p} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n \\ \vec{p}\,' &= \vec{p}\,'_1 + \vec{p}\,'_2 + \vec{p}\,'_3 + \ldots + \vec{p}\,'_n \end{aligned}\right| p = p'$$

3/ THE OTHER LAWS OF NEWTON

The second law of Newton:

"The derivative of momentum is called force."

This means that the resultant of the forces applied to the particle is: $\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$

This equation is called the "**equation of motion**."

In the case of constant mass: following what has just been said, if the mass m of the object is constant (which is common in Newtonian mechanics), then the previous equation is written as:

$$\vec{F} = \frac{d(m\vec{v})}{dt} \Rightarrow \vec{F} = m\frac{d\vec{v}}{dt} \Rightarrow \boxed{\vec{F} = m.\vec{a}}$$

Special case: If the resultant force $\vec{F}$ is constant, then the acceleration $\vec{a} = \frac{\vec{F}}{m}$ is also constant, and the motion is **uniformly varied rectilinearly**.

That's precisely what happens to bodies falling freely under the influence of gravity (referred to as weight): $\vec{P} = m.\vec{g}$

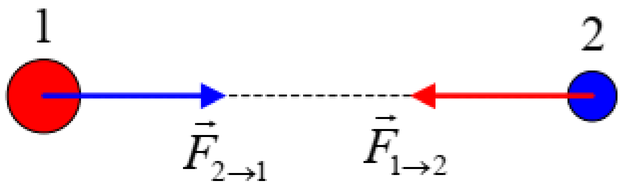
In the case of variable mass: in this scenario, the resultant force $\vec{F}$ is expressed in the form:

$$\vec{F} = \frac{d(m\vec{v})}{dt} \Rightarrow \boxed{\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}.\frac{dm}{dt}}$$

The third law of Newton, or the principle of action and reaction:

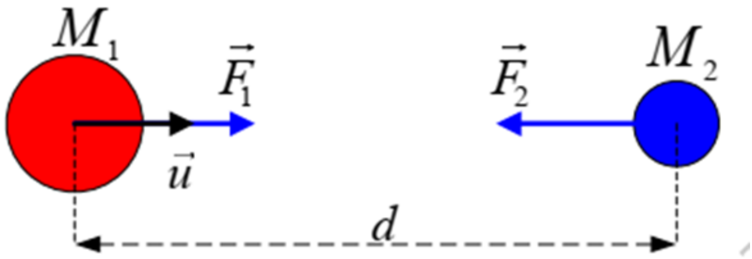
Statement of the law: "**When two particles exert influence on each other, the force applied by the first particle on the second is equal and opposite in direction to the force applied by the second particle on the first**."

This is illustrated in the following figure and allows us to write: $\boxed{\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}}$



4/ LAW OF UNIVERSAL GRAVITATION

The law of universal gravitation, established by Newton in 1685, is the foundation of the theory that explains numerous physical phenomena: from the motion of planets to the free fall of bodies and even the movement of tides.



This law explains the attraction between two bodies of respective masses , separated by the distance d. These two bodies attract each other with two directly opposing forces.

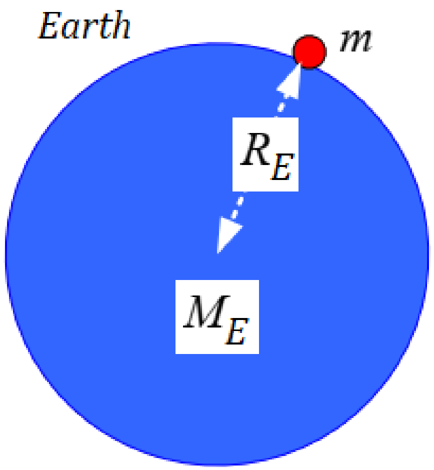
$$\vec{F}_1 = G\frac{M_1.M_2}{d^2}\vec{u} \Rightarrow \boxed{F_1 = G\frac{M_1.M_2}{d^2}}$$

Gravitational Field

The force of terrestrial attraction is the weight. It is customary to calculate weight using the acceleration due to gravity $\vec{g}\ \left(\vec{P} = m\vec{g}\right)$. Thanks to the law of universal gravitation and the law of weight force, we can determine the expression for ***g*** in terms of altitude:

At the surface of the Earth: We obtain the value of the acceleration due to terrestrial gravity as follows:

$$\vec{F} = \vec{P} \Rightarrow G\frac{M_T.m}{R_T^2} = mg_0 \Rightarrow \boxed{g_0 = G\frac{M_T}{R_T^2}} \qquad \boxed{g_0 = 9.8N.kg^{-1}}$$



The universal gravitational constant $G = 6.67 \cdot 10^{-11} N.m^2.kg^{-2}$

Mass of the Earth $M_E = 5.98 \cdot 10^{24} kg$

Radius of the Earth $R_E = 6.37 \cdot 10^6 m$

At height Z above the Earth's surface: The vector of the acceleration due to terrestrial gravity at a height $\boldsymbol{Z}$ from the ground, that is, at the distance
$\boldsymbol{r = R_E + Z}$ from the center of the Earth, is obtained by the following reasoning:
At the surface of the Earth: $$P_0 = mg_0 = G\frac{m.M_E}{R_T^2}$$

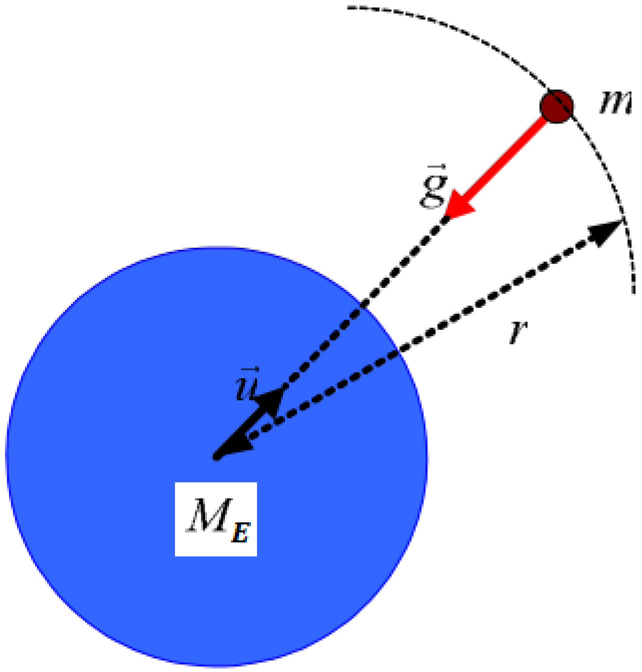
at the distance $\boldsymbol{r}$ from the center of the Earth:

$$P = mg = G\frac{m.M_E}{r^2}$$

$$\boxed{g = g_0\frac{R_T{}^2}{r^2}}$$

As for the vector expression, it is:

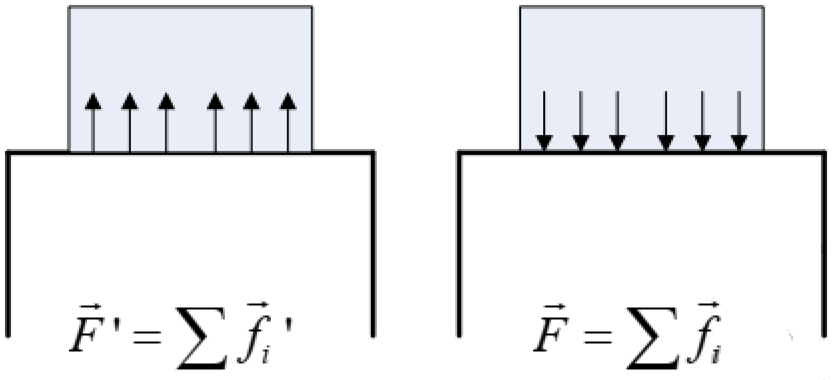
$$\vec{g} = -g_0\frac{R^{\ 2}}{r^2}\vec{u}$$



5/ BINDING FORCES OR CONTACT FORCES

Let's clarify that here we're discussing the forces acting mutually between bodies in contact. The following figure represents a solid body placed on a table. The body is in equilibrium on this table, meaning that the acceleration is zero $\left(\vec{a}=\vec{0}\right)$.

Both forces $\vec{F}$ and $\overrightarrow{F'}$ are called contact or binding forces because of the contact between the two bodies.



6/ FRICTIONAL FORCES

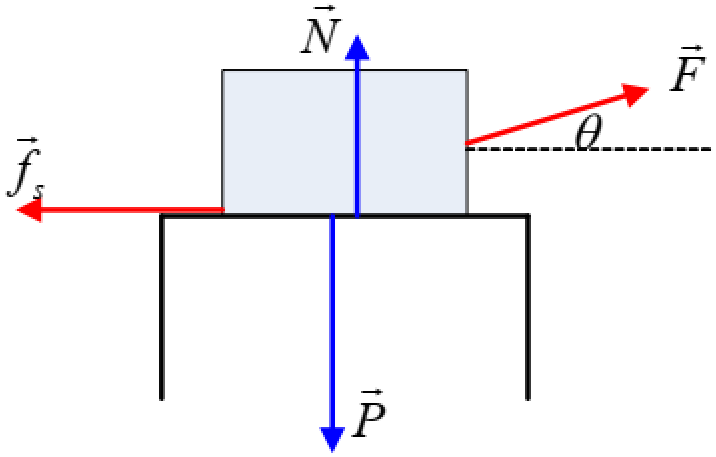
Whenever there's contact between two rough surfaces of solid bodies, a resistance appears and opposes the relative movement of the two bodies. There are several types of friction:

- Frictions between solid bodies, which can be static and dynamic.
- Frictions within fluids.

Static frictional force

The static frictional force is the force that keeps the body at rest even in the presence of an external force.

Case of a body placed on a horizontal plane: Consider the body in the Figure. It is subjected to four forces. Let $\overrightarrow{f_s}$ be the static frictional force. $\vec{P}$ and $\vec{N}$ are respectively the weight and the reaction force. For the body placed on the table to start moving, a minimum force $\vec{F}$ must be applied to it.



The body is at rest: $\sum_i \vec{F_i} = \vec{0}$

By projecting onto the two horizontal and vertical axes, we obtain:

$$\left.\begin{array}{l} N + F.\sin\theta - P = 0 \\ F.\cos\theta - f_s = 0 \end{array}\right| \Rightarrow \boxed{f_s = F.cos\theta}$$

If the angle $\boldsymbol{\theta}$ were zero, we would have $f_s = F$ and $P = N$

Notice that $\boldsymbol{P \neq N}$, with $\boldsymbol{N = P - F \cdot \sin\theta}$, which is the force that keeps the body at rest until the applied force $\vec{\boldsymbol{F}}$ manages to pull it off the surface. Just before pulling the body, the static frictional force reaches its maximum value defined by the law: $\boldsymbol{f_s = \mu_s \,.\, N}$

Where $\boldsymbol{\mu_s}$ is the coefficient of static friction and $\boldsymbol{N}$ is the normal force.

So:

$$\boxed{f_s \leq f_{s\,MAX} = \mu_s \,.\, N}$$

In our example: $N = P - F\sin\theta \Rightarrow$ $\boxed{f_{s\,MAX} = \mu_s \,.\, N = \mu_s\,(P - F\sin\theta)}$

The condition is that $\boldsymbol{N > 0}$ and consequently $\boldsymbol{P > F \cdot \sin\theta}$, otherwise, the body would lift up.

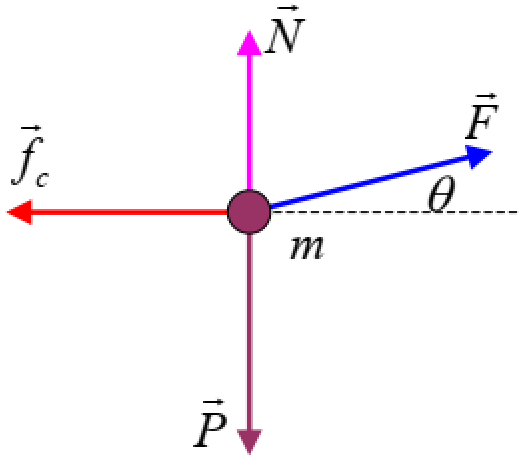
kinetic frictional force

The kinetic frictional force is the force that opposes the motion of a body on a rough surface. Its intensity is given by the formula:

$$\boxed{f_k = \mu_k \,.\, N}$$

Note: In the case of static frictional forces, the body is at rest, whereas in the case of kinetic or dynamic frictional forces, the body is in motion.

Let's consider the example schematized in the figure.
The body is now considered to be in motion.
It's possible to determine the expression for dynamic frictional force after establishing the expression for the normal force:



$$\left.\begin{aligned} &\boldsymbol{N = P - F \cdot \sin\theta} \\ &\boldsymbol{f_k = \mu_k \, . \, N} \end{aligned}\right| \Rightarrow \boxed{\boldsymbol{f_k = \mu_k \, (P - F \sin\theta)}}$$

By applying the fundamental relation of dynamics, where is ***m*** the mass of the body, we can write:

$$\boldsymbol{F \cos\theta - f_k = m\, a} \;\Rightarrow\; \boxed{\boldsymbol{f_k = F \cos\theta - m\, a}}$$

Where $\boldsymbol{\mu_k}$ is the symbol for the coefficient of kinetic (or dynamic) friction, ***N*** and represents the normal force.
Here we are not discussing the maximum frictional force

Friction in fluids

When a solid body moves within a fluid (gas or liquid), a frictional force appears. It is calculated by the formula: $\boxed{\overrightarrow{f_f} = -K\eta \cdot \vec{v}}$

K: A coefficient depending on the shape of the solid body moving within the fluid.

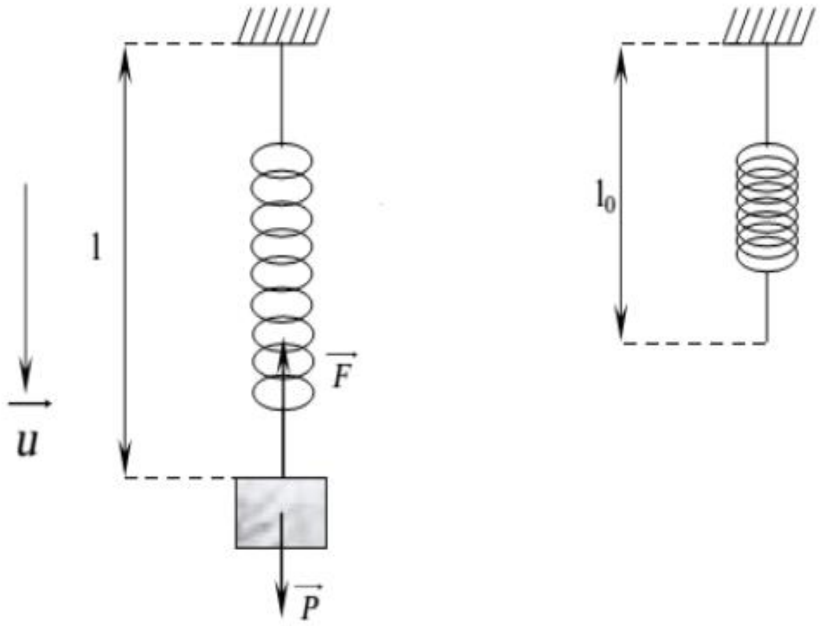
η: A coefficient depending on internal friction within the fluid. Internal friction within the fluid is called **viscosity**.

7/ ELASTIC FORCES

Elastic forces cause periodic motions.

The simplest example is the spring's restoring force.

$$\vec{F} = -k(l - l_0)\vec{u}$$



By projection onto the axis, we arrive at the following force law: $\boxed{F = -kx}$

k: coefficient of elongation

(stiffness coefficient of the spring)