

Homework (Devoir maison)    December 2024

\*\* (Utiliser les notations du cours)  
 \*\* (On tiendra compte de la présentation des copies)

**Exercise 1.** (1) Give the definition of : – a regular distribution, – a singular distribution - the norm in  $\mathcal{D}(\mathbb{R}^n)$ , – the derivation in  $\mathcal{D}'(\mathbb{R}^n)$ , – the convolution in  $\mathcal{D}'(\mathbb{R})$ , – the distributions  $\text{pv}\frac{1}{2-x}$ ,  $H$  and  $\delta_{x=1}$  in  $\mathcal{D}'(\mathbb{R})$ .

**Exercise 2.** Prove that  $\text{pf}\frac{H}{x^2} \in \mathcal{D}'(\mathbb{R})$  :

$$\langle \text{pf}\frac{H}{x^2}, \varphi \rangle = \lim_{\epsilon \downarrow 0} \left\{ \int_{\epsilon}^{\infty} \frac{\varphi(x)}{x^2} dx - \frac{\varphi(0)}{\epsilon} + \varphi'(0) \log \epsilon \right\}, \quad \forall \varphi \in \mathcal{D}(\mathbb{R}).$$

Calculate  $(\text{pf}\frac{H}{x^2})'$ . In the same way, define  $\text{pf}\frac{H}{x^3}$  in  $\mathcal{D}'(\mathbb{R})$  and calculate  $(\text{pf}\frac{H}{x^3})'$ .

**Exercise 3.** Let  $T(x) := (\sin x)\delta''$ . Prove that  $T \in \mathcal{D}'(\mathbb{R})$ . Calculate  $T'$ .

**Exercise 4.** Let

$$\langle T, \varphi \rangle = \int_{-\infty}^{\infty} \varphi(2x + 3, -x) dx, \quad \forall \varphi \in \mathcal{D}(\mathbb{R}^2).$$

Prove that  $T \in \mathcal{D}'(\mathbb{R}^2)$ , and calculate  $(\partial_x - \partial_y)T$  in  $\mathcal{D}'(\mathbb{R}^2)$ .

**Exercise 5.** Let  $f(x) := \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

Calculate  $f * f, \quad f' * f''$ .

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