

Homework (Devoir maison) December 2024

- ** (Utiliser les notations du cours)
- ** (On tiendra compte de la présentation des copies)

Exercise 1. Give the definition of :

- (i) the functions ρ et γ such that $\rho(\xi) + \sum_{j=1}^{\infty} \gamma(2^{-j}\xi) = 1 \quad (\forall \xi \in \mathbb{R})$,
- (ii) the operators S_j et Q_j ,
- (iii) the spaces $\mathcal{S}_{\infty}(\mathbb{R})$, $\mathcal{S}'_{\infty}(\mathbb{R})$, $B_{p,q}^s(\mathbb{R})$ and $\dot{B}_{p,q}^s(\mathbb{R})$.
- (iv) Give examples of functions in $\mathcal{S}'_{\infty}(\mathbb{R})$.

Exercise 2. Prove that, if $s > 0$ then

$$B_{p,q}^s(\mathbb{R}) = L_p(\mathbb{R}) \cap \dot{B}_{p,q}^s(\mathbb{R}).$$

Exercise 3. 1- Show that $\dot{B}_{p,q}^s(\mathbb{R})$ is a Banach space.

2- Show that BMO is a Banach space modulo constants.

3- Prove that $\log |x| \in BMO$. Find a conclusion ?

Exercise 4. Let $\theta \in \mathcal{D}(\mathbb{R}^n \setminus \{0\})$ be such that $\text{supp } \theta \subset \{\xi \in \mathbb{R}^n : a \leq |\xi| \leq b\}$, $0 < a < b$. Prove that

$$\theta(2^{-j}\xi) = \sum_{k=j+m_1}^{j+m_2} \theta(2^{-j}\xi)\gamma(2^{-k}\xi).$$

Find the numbers m_1 and m_2 .

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